

4 Rounding and Estimating

4.1 Revision of the Four Rules: Whole Numbers



Example 1

Calculate:

(a) $464 + 97$

(b) $184 - 36$

(c) 47×12

(d) $710 \div 5$



Solution

(a)
$$\begin{array}{r} 464 \\ + 97 \\ \hline 561 \\ 11 \end{array}$$

(b)
$$\begin{array}{r} 71 \\ 184 \\ - 36 \\ \hline 148 \end{array}$$

(c)
$$\begin{array}{r} 47 \\ \times 12 \\ \hline 94 \\ 470 \\ \hline 564 \\ 1 \end{array}$$

(d)
$$5 \overline{)142} \begin{array}{l} 28 \\ \underline{14} \\ 20 \\ \underline{15} \\ 50 \\ \underline{50} \\ 0 \end{array}$$



Exercises

Work out the answer to each question *without* using a calculator. Check your answers with a calculator.

1. (a) $13 + 16$

(b) $24 + 22$

(c) $45 + 34$

(d) $123 + 51$

(e) $214 + 135$

(f) $201 + 356$

2. (a) $36 + 102$

(b) $88 + 35$

(c) $66 + 282$

(d) $97 + 142$

(e) $361 + 421$

(f) $188 + 924$

3. (a) $25 - 13$

(b) $66 - 22$

(c) $97 - 46$

(d) $136 - 121$

(e) $258 - 39$

(f) $971 - 420$

4. (a) $199 - 42$ (b) $643 - 132$ (c) $198 - 156$
 (d) $372 - 184$ (e) $924 - 138$ (f) $3631 - 179$
5. (a) 12×3 (b) 11×5 (c) 23×2
 (d) 31×3 (e) 22×4 (f) 101×6
6. (a) 19×5 (b) 86×4 (c) 39×6
 (d) 27×7 (e) 43×9 (f) 65×8
7. (a) 82×11 (b) 37×12 (c) 39×42
 (d) 54×23 (e) 61×34 (f) 87×65
8. (a) $68 \div 2$ (b) $64 \div 4$ (c) $123 \div 3$
 (d) $845 \div 5$ (e) $312 \div 6$ (f) $1407 \div 7$
9. (a) $240 \div 20$ (b) $720 \div 12$ (c) $880 \div 44$
 (d) $630 \div 15$ (e) $750 \div 25$ (f) $345 \div 23$
10. (a) 87×3 (b) $192 + 249$ (c) $186 - 95$
 (d) 36×43 (e) $915 \div 5$ (f) 48×17

4.2 Revision of the Four Rules: Decimals



Example 1

Calculate:

- (a) $3.8 + 10.42$ (b) $18.2 - 0.36$
 (c) 8.2×3.7 (d) $1.56 \div 0.3$



Solution

$$\begin{array}{r} \text{(a)} \quad 3.8 \\ + 10.42 \\ \hline 14.22 \\ \hline 1 \end{array}$$

$$\begin{array}{r} \text{(b)} \quad \overset{7}{1} \overset{1}{8} \overset{1}{.} 20 \\ - 0.36 \\ \hline 17.84 \end{array}$$

$$\begin{array}{r}
 \text{(c)} \quad 8.2 \\
 \times 3.7 \\
 \hline
 574 \\
 2460 \\
 \hline
 30.34 \\
 \hline
 1
 \end{array}$$

$$\text{(d)} \quad 1.56 \div 0.3 = 15.6 \div 3$$

$$\begin{array}{r}
 5.2 \\
 3 \overline{)15.6} \\
 \hline
 0 \\
 6 \\
 6 \\
 0 \\
 6 \\
 6 \\
 0
 \end{array}$$



Exercises

Solve each of the following *without* using a calculator. Check your answers with a calculator.

- | | | | |
|----|-----------------------|-----------------------|-----------------------|
| 1. | (a) $3.5 + 4.2$ | (b) $16.1 + 32.6$ | (c) $1.5 + 3.8$ |
| | (d) $13.3 + 4.61$ | (e) $18.6 + 0.42$ | (f) $3.14 + 0.612$ |
| 2. | (a) $6.4 - 2.1$ | (b) $27.8 - 13.6$ | (c) $3.2 - 0.8$ |
| | (d) $8.2 - 4.5$ | (e) $6.62 - 0.34$ | (f) $8.3 - 6.27$ |
| 3. | (a) 4.3×2 | (b) 3.5×4 | (c) 7.4×6 |
| | (d) 6.2×7 | (e) 18.3×9 | (f) 5.62×5 |
| 4. | (a) $6.8 \div 2$ | (b) $63.9 \div 3$ | (c) $52.4 \div 4$ |
| | (d) $75.5 \div 5$ | (e) $99.4 \div 7$ | (f) $151.8 \div 6$ |
| 5. | (a) $12.6 + 8.5$ | (b) $76.3 - 18.7$ | (c) $20.39 - 15.6$ |
| | (d) 17.6×4 | (e) 132.7×6 | (f) $36.61 \div 7$ |
| 6. | (a) 5.6×0.3 | (b) 2.3×1.5 | (c) 4.8×0.21 |
| | (d) 3.4×9.4 | (e) 3.6×0.72 | (f) 8.2×0.91 |
| 7. | (a) $18.6 \div 0.3$ | (b) $74.5 \div 0.5$ | (c) $0.36 \div 0.02$ |
| | (d) $10.5 \div 5$ | (e) $45 \div 0.09$ | (f) $0.84 \div 0.4$ |
| 8. | (a) $21.6 \div 0.4$ | (b) $8.2 - 0.37$ | (c) 0.62×7 |
| | (d) 3.2×0.17 | (e) $8.4 \div 8$ | (f) 3.7×2.01 |

4.3 Order of Operations

Brackets

O

Division

Multiplication

Addition

Subtraction

BODMAS can be used to remember the order in which to carry out operations



Example 1

Calculate:

(a) $(3 + 2) \times 6 - 8$

(b) $4 \times 6 + 18 \div 2$

(c) $(17 - 2) \div 5 + 6$



Solution

(a) $(3 + 2) \times 6 - 8$

(brackets first)

$= 5 \times 6 - 8$

(multiplication second)

$= 30 - 8$

(subtraction last)

$= 22$

(b) $4 \times 6 + 18 \div 2$

(multiplication and division must be done before addition)

$= 24 + 9$

$= 33$

(c) $(17 - 2) \div 5 + 6$

(brackets first)

$= 15 \div 5 + 6$

(division second)

$= 3 + 6$

(addition last)

$= 9$



Example 2

State whether each one of the statements below is *true* or *false*:

(a) $3 + 6 \times 2 = 15$

(b) $30 - 7 \times 4 = 92$

(c) $8 + 20 \div 2 = 14$



Solution

$$(a) \quad 3 + 6 \times 2 = 3 + 12 \quad (\text{multiplication must be done before addition})$$

$$= 15$$

Therefore the statement is *true*.

$$(b) \quad 30 - 7 \times 4 = 30 - 28 \quad (\text{multiplication must be done before subtraction})$$

$$= 2$$

Therefore the statement is *false*.

$$(c) \quad 8 + 20 \div 2 = 8 + 10 \quad (\text{division must be done before addition})$$

$$= 18$$

Therefore the statement is *false*.



Exercises

1. Calculate:

$$(a) \quad 6 + 7 \times 2$$

$$(b) \quad 8 - 3 \times 2$$

$$(c) \quad 19 - 4 \times 3$$

$$(d) \quad 3 \times 6 - 9$$

$$(e) \quad 15 - 4 + 7 \times 2$$

$$(f) \quad 11 \times 3 + 2$$

$$(g) \quad 16 \times 4 - 3$$

$$(h) \quad 6 + 7 \times 2 - 20 \div 4$$

$$(i) \quad 18 \times 2 - (4 + 7)$$

$$(j) \quad 16 - 5 \times 2 + 3$$

2. State whether each one of the statements below is *true* or *false*. Calculate the correct answer for those that are *false*.

$$(a) \quad 6 \times 7 - 2 = 40$$

$$(b) \quad 8 \times (6 - 2) + 3 = 56$$

$$(c) \quad 35 - 7 \times 2 = 56$$

$$(d) \quad 3 + 7 \times 3 = 30$$

$$(e) \quad 18 - (4 + 7) = 21$$

$$(f) \quad 43 - 3 + 2 = 42$$

$$(g) \quad 80 \div 2 + 6 = 10$$

$$(h) \quad 64 - 10 + 2 = 52$$

3. Put brackets into each of the statements below to make it correct:

$$(a) \quad 3 \times 6 + 1 = 21$$

$$(b) \quad 5 + 6 \times 2 = 22$$

$$(c) \quad 45 \div 6 + 3 = 5$$

$$(d) \quad 49 - 3 + 2 = 44$$

$$(e) \quad 7 \times 3 + 2 = 35$$

$$(f) \quad 13 - 4 \times 2 = 18$$

4. Write out each of the calculations below, filling in the missing numbers:

(a) $3 \times ? + 2 = 17$

(b) $? \times 5 - 8 = 22$

(c) $(4 + ?) \times 2 = 20$

(d) $6 - ? \times 2 = 0$

(e) $(7 - ?) \times 4 = 20$

(f) $? \div 3 + 4 = 8$

5. Jane writes down:

$$4 \times 7 + 2 \times 3 = 90$$

(a) Explain why her answer is incorrect, and calculate the correct answer.

(b) By using brackets Jane can make her calculation correct. Show how this can be done.

6. Esther and Andy are given this problem:

$$30 \div 6 - 3 + 1$$

Esther says the answer is 1.

Andy says the answer is 11.

(a) Is either of them correct?

(b) Show how Esther could insert brackets to give her answer.

(c) Show how Andy could insert brackets to give his answer.

7. State whether each one of the statements below is *true* or *false*:

(a) $(3 \times 6) \times 2 = 3 \times (6 \times 2)$

(b) $(4 + 2) + 7 = 4 + (2 + 7)$

(c) $(8 - 2) - 1 = 8 - (2 - 1)$

(d) $(8 \div 2) \div 2 = 8 \div (2 \div 2)$

8. Put brackets into each of the calculations below to make it correct:

(a) $13 - 4 - 1 = 10$

(b) $30 - 9 + 2 = 19$

(c) $60 \div 6 \div 3 = 30$

9. Calculate:

(a) $8.2 \div 0.2 - 0.1$

(b) $3.6 \times 0.2 - 0.1$

(c) $8.2 \times (6 - 5.4)$

(d) $2.2 - 0.7 \times 0.2$

10. Write out each of the calculations below, filling in the missing numbers:

(a) $0.8 + ? \times 0.6 = 3.2$

(b) $? \times 0.5 + 6 \times 0.4 = 3.9$

(c) $0.9 + 4.8 \div ? = 6.9$

(d) $2.7 \div ? - 1.4 = 1.6$



Exercises

In these questions, do not use a calculator, and remember to show all of your working.

- Tickets for a school party cost £1.25 each. Calculate the cost of:
 - 3 tickets,
 - 14 tickets.
- CDs cost £9 each in a music shop sale. How many CDs can you buy if you have £48 to spend? How much money will you have left over?
- A school buys 30 calculators costing £6.99 each. What is the total cost of these calculators? How could you do this calculation in your head?
- A school mathematics department has £300 to spend on new textbooks. The textbooks cost £7 each. How many books can be bought?
- Prakesh is paid travelling expenses every time he drives his car for work. He is paid £12 for each journey, plus 14p per mile travelled. How much is he paid for:
 - a 50 mile journey,
 - an 82 mile journey?
- How many minibuses, each seating 17 pupils are needed to transport 110 pupils?
- Joanne buys 3 magazines that cost £1.50, £2.45 and 80p. She pays for them with a £10 note. How much change should she get?
- Ben orders 25 floppy discs for his computer. The discs cost 40p each and he has to pay £3.25 postage. How much does he have to pay in total?
- A farmer packs his free-range eggs into boxes that each contain half a dozen eggs. One day he collects 119 eggs. How many boxes can he fill and how many eggs does he have left over?
- Alison buys 6 tapes that cost £8.99 each. She pays for them with three £20 notes. How much change should she get?

4.5 Rounding

We *round* numbers when all we need is a *reasonable approximation* rather than the exact value.

The number 8.4236 can be rounded to a specified number of *decimal places*:

8.4236 → 8.424 to 3 decimal places

8.4236 → 8.42 to 2 decimal places

8.4236 → 8.4 to 1 decimal place

The number 173.265 can be rounded to a specified number of *significant figures*:

$$173.265 \rightarrow 173.27 \text{ to } 5 \text{ significant figures}$$

$$173.265 \rightarrow 173 \text{ to } 3 \text{ significant figures}$$

$$173.265 \rightarrow 200 \text{ to } 1 \text{ significant figure}$$

A particular digit in a number will *round up* if the digit that follows it is 5, 6, 7, 8 or 9.

For example, $52.3\overset{\cdot}{6}8 = 52.37$ (to 2 decimal places).

The digit will *remain unchanged* if the following digit is 0, 1, 2, 3 or 4.

For example, $6.7\overset{\cdot}{4}3 = 6.74$ (to 3 significant figures).



Example 1

Round 3647.5 to the nearest:

- (a) whole number, (b) ten (10),
 (c) hundred (100), (d) thousand (1000).



Solution

- (a) 3648 Note that the 7 rounds up to 8, because it is followed by a 5.
 (b) 3650 Note that the 4 rounds up to a 5, because it is followed by a 7.
 (c) 3600 Note that the 6 is unchanged, because the digit following it in the number is less than 5.
 (d) 4000 Note that the 3 rounds up to a 4, because it is followed by a 6.



Example 2

Write 13.68952 correct to:

- (a) 1 decimal place, (b) 3 decimal places, (c) 2 decimal places.



Solution

- (a) 13.7 to 1 decimal place.
 (b) 13.690 to 3 decimal places.
 (c) 13.69 to 2 decimal places.

To make sure that you obtain the correct answer when you use a calculator,

ESTIMATE the answer mentally,

CALCULATE the answer using your calculator
and then

CHECK that the calculator answer is sensible
by comparing it with your mental estimate.



Example 2

Halim uses his calculator to work out 8.623×4.71 .

He gets the answer 406.1433.

Use an estimate to check his answer.



Solution

$$\begin{aligned} \text{(a) Estimate} &= 9 \times 5 \\ &= 45 \end{aligned}$$

Halim's answer should have been 40.61433.



Example 3

Jai carries out the following calculations on his calculator, and writes his answers correct to 3 decimal places:

A $3.62 \times 8.94 = 32.363$

B $47.92 \div 2.17 = 1.512$

C $184 \times 3.616 = 665.344$

D $(21.4 + 19.7) \times 3.61 = 14.837$

Use estimates to decide which answers *could be correct* and which are *definitely incorrect*.



Solution

$$\begin{aligned} \text{A Estimate} &= 4 \times 9 \\ &= 36 \end{aligned}$$

suggesting that Jai's answer *could be correct*.

$$\begin{aligned} \text{B Estimate} &= 50 \div 2 \\ &= 25 \end{aligned}$$

showing that Jai's answer *must be incorrect*.

$$\begin{aligned} \text{C} \quad \text{Estimate} &= 200 \times 4 \\ &= 800 \end{aligned}$$

suggesting that Jai's answer *could be correct*.

$$\begin{aligned} \text{D} \quad \text{Estimate} &= (20 + 20) \times 4 \\ &= 40 \times 4 \\ &= 160 \end{aligned}$$

showing that Jai's answer *must be incorrect*.



Exercises

- For each of the calculations listed below,
 - estimate* the answer,
 - use a calculator to *work out* the answer,
 - compare* your estimate with the answer from the calculator:

(a) 4.7×8.34	(b) 9.6×21.43	(c) 11.46×8.02
(d) 18.3×108	(e) 95×76	(f) 15.4×24.9
- Boxes of matches each contain 52 matches. Estimate the total number of matches in 8 boxes.
- The floor of a room measures 3.61 m by 4.72 m.
 - Estimate* the area of the floor of the room.
 - Calculate* the area of the floor, using a calculator.
 - Compare* your estimate with the answer from the calculator.
- Estimate the cost of 23 cans of drink costing 37p each.
- Kyle uses his calculator to do the calculations listed below, and gives his answers correct to 3 decimal places:

A	$36.41 \times 37.32 = 135.882$
B	$56.2 \times 1.97 = 11.071$
C	$82.3 \times 0.625 = 51.438$
D	$(204 + 109) \times 10.2 = 3.193$
E	$(16.7 + 31.3) \div 4.75 = 1.011$

By using estimates, decide which calculations Kyle has *not* done correctly.

6. Make estimates for each of the calculations below:

(a) $\frac{6.1 \times 3.4}{4.2}$

(b) $\frac{7.3 + 9.1}{2.3}$

(c) $\frac{62.6 \times 21.3}{34.9}$

(d) $\frac{71.3 \times 99.6}{11.3}$

(e) $\frac{142.3 - 93.6}{23.8}$

(f) $\frac{16.5 \times 19.2}{33.6 - 21.9}$

7. Estimate the cost of 18 calculators that cost £7.99 each.

8. A can contains 330 ml of a drink and there are 144 cans in a box. Estimate the total volume of drink in a box, in litres.

9. A car uses 0.18 litres of fuel to travel 1 mile.

(a) *Estimate* the amount of fuel that is used on a 162 mile journey.

(b) *Use a calculator* to work out the amount of fuel that is used.

(c) Does your estimate support the answer from your calculator?

10. Tickets to watch a football match cost £19 each. If 26 472 people watch the match, estimate the total amount that has been paid by these spectators.

4.7 Calculator Logic - Bracket and Memory Keys

When using your calculator it is important to be aware of both how it works and how to make the best use of it. Most calculators have *bracket* and *memory* keys that can be used for more complex calculations.

Note: A *scientific calculator* will always try to apply the rules of BODMAS.

Brackets can be inserted at any stage of a calculation by using the bracket keys $($

and $)$. The calculator may show an error message if brackets are not in pairs.

The notation on calculator memory keys varies from one machine to another. You need to find out the keys on your calculator that perform the following functions:

$M\ in$ or STO places a number on display in the memory.

$M\ +$ adds the number on display to the number in the memory.

$M -$ subtracts the number on display from the number in the memory.

$M R$ or RCL brings the number in the memory to the display screen.

Some of these keys perform other functions in other modes (especially in statistical mode).

One thing you will need to find out for yourself is how to *empty* the contents of the memory; this varies from one calculator to another.



Example 1

Calculate

$$\frac{6.2 + 8.6}{3.9 - 2.4}$$

correct to 3 significant figures.



Solution

Using brackets,

$((6.2 + 8.6) \div (3.9 - 2.4)) =$ gives 9.87 to 3 significant figures

Using the memory,

$3.9 - 2.4 = M \text{ in}$
 $6.2 + 8.6 = \div M R =$ gives 9.87 to 3 significant figures



Example 2

Calculate

$$\frac{6}{3 + 4 \times 7.2}$$

correct to 2 decimal places.



Solution

Using brackets,

$6 \div ((3 + 4 \times 7.2)) =$ gives 0.1886... = 0.19 to 2 decimal places

(Remember that a scientific calculator will apply BODMAS.)

Using memory,

$$3 \text{ (+) } 4 \text{ (\times) } 7.2 \text{ (=) } \text{(M in)}$$

$$6 \text{ (\div) } \text{(MR)} \text{ (=) } \text{ gives } 0.1886 \dots$$

$$= 0.19 \text{ to 2 decimal places}$$



Example 3

Do you need to include brackets if you use a scientific calculator to work out:

(a) $3 \times 4 + 6 \times 2$

(b) $\frac{24}{8-2}$?



Solution

(a) The correct answer is $3 \times 4 + 6 \times 2 = 12 + 12$

$$= 24$$

Using a scientific calculator without brackets also gives 24, so brackets *are not* needed here.

(b) The correct answer is $\frac{24}{8-2} = \frac{24}{6}$
 $= 4$

Without brackets the calculator gives the answer 1.

It actually works out $24 \div 8 - 2$ or $\frac{24}{8} - 2$ which gives 1, so brackets *are* needed here.



Exercises

1. Carry out the following calculations using a calculator, giving your answers, where necessary, correct to 2 decimal places:

(a) $6 \times (8.7 - 1.05)$

(b) $\frac{2 \times 47}{6 + 9}$

(c) $\frac{6 + 17}{3}$

(d) $\frac{42 - 3}{7}$

(e) $\frac{6 + 22}{47 - 21}$

(f) $\frac{9 - 32}{8 - 27}$

2. Carry out the calculation below. You may use the memory of your calculator, but not the bracket keys. Give your answers correct to 3 significant figures.

(a) $\frac{4.9}{3.7 \times 2.6}$

(b) $\frac{4.7}{16 - 7}$

(c) $\frac{9.2 \times 6.7}{4 + 16.2}$

(d) $\frac{11.2 - 9.47}{12 - 0.81}$

3. Use brackets or the memory facilities on a calculator to calculate the following, giving your answers, where necessary, correct to 3 decimal places.

(a) $8 + \frac{6}{9 + 7}$

(b) $\frac{1.9 + 12.2}{8 - 3}$

(c) $\frac{6.3 \times 5.32 + 6.49}{(2.94 - 1.62) \times 3.5}$

(d) $\frac{21.5}{8 + 3} + \frac{6.7 + 3.2}{4.9}$

4. Calculate

$$\frac{4.7 \times (5.32 + 6.49)}{(2.94 - 1.62) \times 3.5}$$

- (a) correct to 2 significant figures,
 (b) correct to 2 decimal places.

5. James tried to calculate $\frac{6}{8 + 2}$. He obtained the answer 2.75, which is wrong.

- (a) What is the correct answer?
 (b) What did James do wrong?

6. Do you need to use brackets if you work the calculations below out on a calculator, *without* using the memory facilities?

(a) $\frac{6}{2} + \frac{9}{5}$

(b) $\frac{8}{4} + 3 \times 6$

(c) $\frac{6 + 9}{2}$

(d) $\frac{8 + 3 \times 6}{4}$

7. Calculate $\frac{3 + 9 + 17 + 8 + 6 + 9 + 4 + 7}{5 + 3}$ using a calculator,

giving your answer correct to:

- (a) 2 decimal places, (b) 2 significant figures.

8. For each set of instructions given below, write down the calculation that it was used to find:

(a) $(6 \div (8 - 7) + 2) =$

(b) $(4 + 7) \times (9 + 4) =$

(c) $4 \div (8 - 5) + 6 =$

(d) $(1 + 7) \times (8 - 3) \div (2 + 9) =$

9. Without using the fraction key, use your calculator to work out the following, giving your answers, where necessary, correct to 3 significant figures:

(a) $\frac{1}{6} + \frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2} + 1$

(b) $\frac{1}{\frac{1}{6} + \frac{1}{5} + \frac{1}{4} + \frac{1}{3}}$

(c) $\frac{1}{1 + \left(\frac{1}{\left(6 + \frac{1}{5} \right)} \right)}$